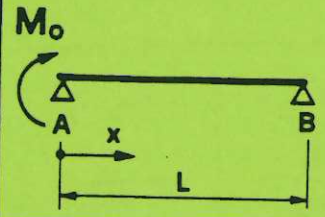
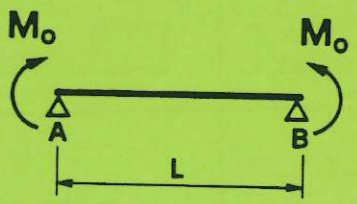
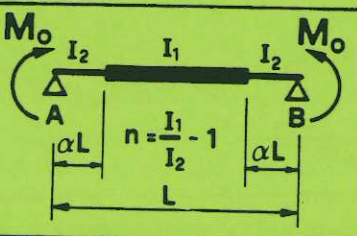
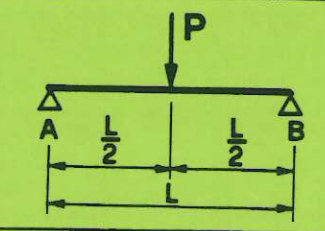
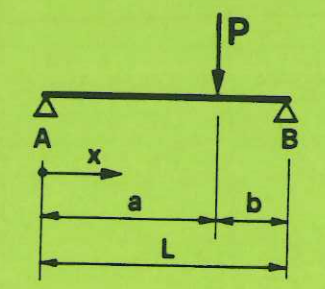
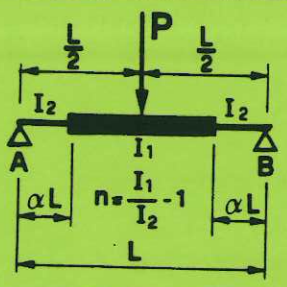
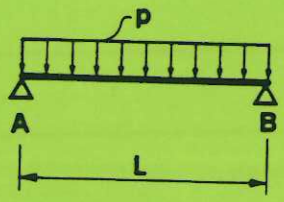
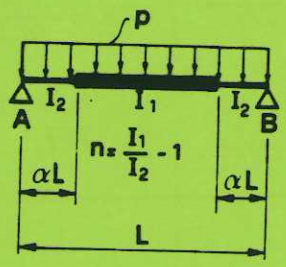
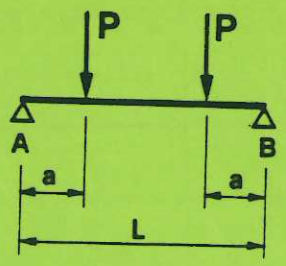
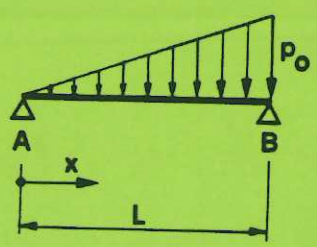
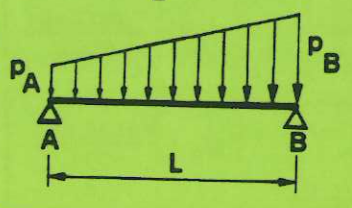
RÉACTIONS D'APPUI, MOMENTS FLÉCHISSANTS, FLÈCHES ET ANGLES DE ROTATION DES POUTRES À UNE TRAVÉE

Convention de signe

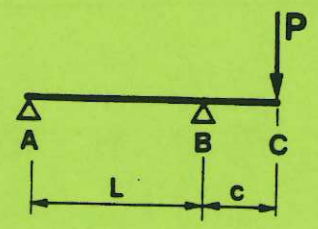
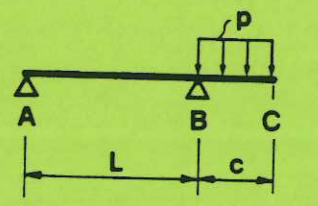
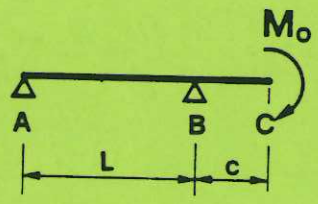
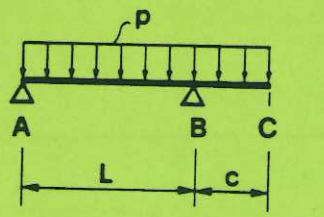
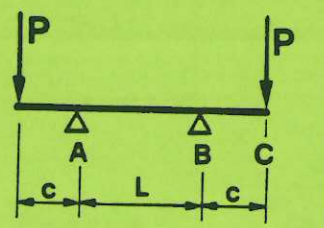
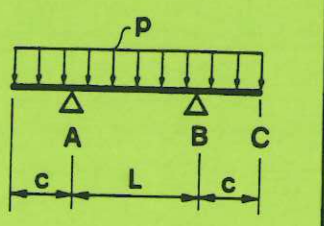
Réaction d'appui R : positive vers le haut
Moment fléchissant M : positif s'il tend les fibres inférieures
Flèche f : positive vers le bas
Angle de rotation φ : positif dans le sens trigonométrique

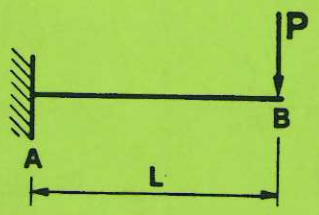
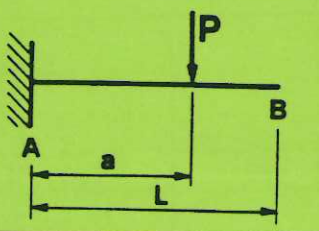
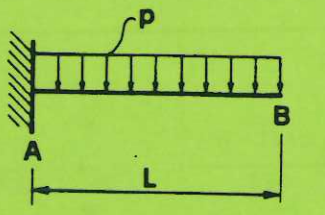
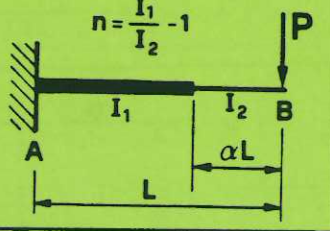
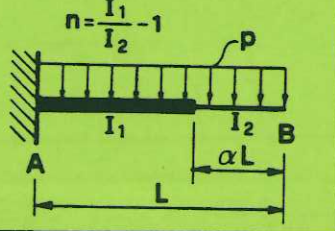
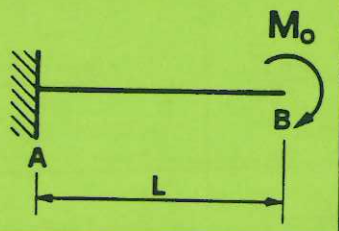
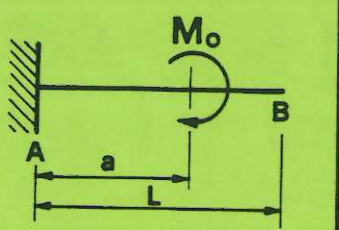
POUTRE À DEUX APPUIS

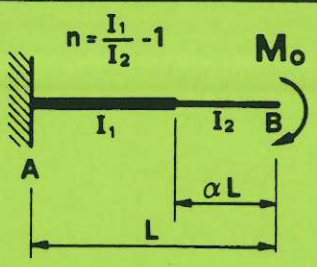
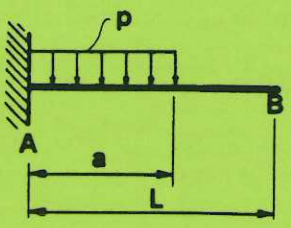
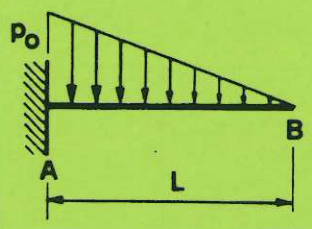
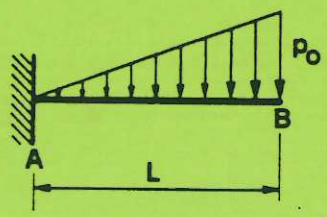
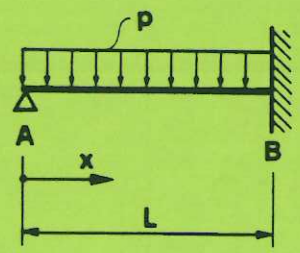
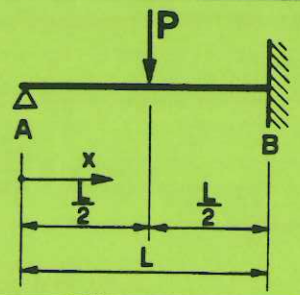
Cas de charge	Réactions d'appui	Moments fléchissants	Flèches	Angles de rotation
	$-R_A=R_B=\frac{M_0}{L}$	$M_A=M_0$	$f_{max}=\frac{M_0L^2}{9\sqrt{3}EI}$ pour $x=L-\frac{L}{\sqrt{3}}$ $f_{(\frac{L}{2})}=\frac{M_0L^2}{16EI}$	$\varphi_A=-\frac{M_0L}{3EI}$ $\varphi_B=\frac{M_0L}{6EI}=-\frac{\varphi_A}{2}$
	$R_A=R_B=0$	$M_A=M_B=M_{(\frac{L}{2})}=M_0$	$f_{max}=f_{(\frac{L}{2})}=\frac{M_0L^2}{8EI}$	$-\varphi_A=\varphi_B=\frac{M_0L}{2EI}$
	$R_A=R_B=0$	$M_A=M_B=M_{(\frac{L}{2})}=M_0$	$f_{max}=f_{(\frac{L}{2})}=$ $=\frac{M_0L^2}{8EI_1}(1+4\alpha^2n)$	$-\varphi_A=\varphi_B=$ $=\frac{M_0L}{2EI_1}(1+2\alpha n)$
	$R_A=R_B=\frac{P}{2}$	$M_{max}=M_P=\frac{PL}{4}$	$f_{max}=f_P=\frac{PL^3}{48EI}$	$-\varphi_A=\varphi_B=\frac{PL^2}{16EI}$
	$R_A=\frac{Pb}{L}$ $R_B=\frac{Pa}{L}$	$M_{max}=M_P=\frac{Pab}{L}$	$f_{max}=\frac{Pb(L^2-b^2)}{9EIL}\times$ $\times\sqrt{\frac{L^2-b^2}{3}}$ pour a>b, en x= $=\sqrt{\frac{1}{3}(L^2-b^2)}$ $f_P=\frac{Pa^2b^2}{3EIL}$	$\varphi_A=-\frac{Pab(L+b)}{6EIL}$ $\varphi_B=\frac{Pab(L+a)}{6EIL}$

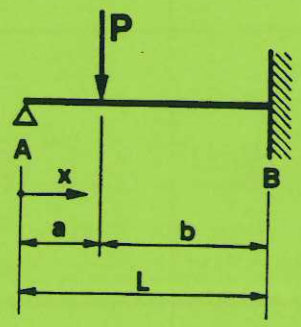
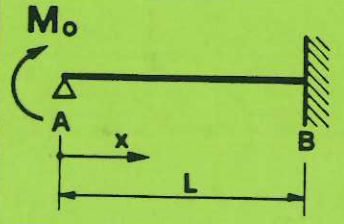
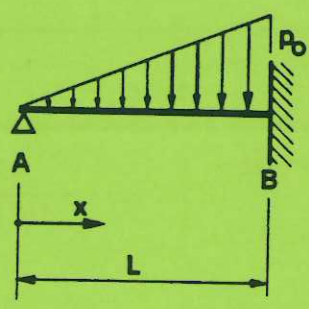
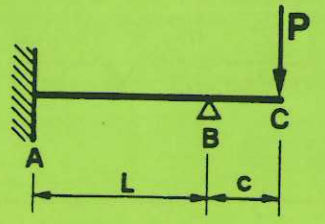
Cas de charge	Réactions d'appui	Moments fléchissants	Flèches	Angles de rotation
	$R_A=R_B=\frac{P}{2}$	$M_{max}=M_P=\frac{PL}{4}$	$f_{max}=f_P=$ $=\frac{PL^3}{48EI_1}(1+8\alpha^3 n)$	$-\varphi_A=\varphi_B=$ $=\frac{PL^2}{16EI_1}(1+4\alpha^2 n)$
	$R_A=R_B=\frac{pL}{2}$	$M_{max}=M_{(\frac{L}{2})}=\frac{pL^2}{8}$	$f_{max}=f_{(\frac{L}{2})}=$ $=\frac{5pL^4}{384EI}$	$-\varphi_A=\varphi_B=\frac{pL^3}{24EI}$
	$R_A=R_B=\frac{pL}{2}$	$M_{max}=M_{(\frac{L}{2})}=\frac{pL^2}{8}$	$f_{max}=f_{(\frac{L}{2})}=$ $=\frac{5pL^4}{384EI_1}\times$ $\times(1+12,8\alpha^3 n-$ $-9,6\alpha^4 n)$	$-\varphi_A=\varphi_B=$ $=\frac{pL^3}{24EI_1}\times$ $\times(1+6\alpha^2 n-$ $-4\alpha^3 n)$
	$R_A=R_B=P$	$M_{max}=M_P=Pa$	$f_P=\frac{Pa^2(3L-4a)}{6EI}$ $f_{max}=f_{(\frac{L}{2})}=$ $=\frac{Pa(3L^2-4a^2)}{24EI}$	$-\varphi_A=\varphi_B=$ $=\frac{Pa(L-a)}{2EI}$
	$R_A=\frac{p_0 L}{6}$ $R_B=\frac{p_0 L}{3}$	$M_{max}=\frac{p_0 L^2}{9\sqrt{3}}$ en $x=\frac{L}{\sqrt{3}}$	$f_{max}=\frac{p_0 L^4}{153,4EI}=$ $=0,00652\frac{p_0 L^4}{EI}$ en $x=0,51933L$	$\varphi_A=-\frac{7p_0 L^3}{360EI}$ $\varphi_B=\frac{8p_0 L^3}{360EI}=$ $=\frac{p_0 L^3}{45EI}$
$P=\frac{p_A+p_B}{2}L$ 	$R_A=\frac{L}{6}(2p_A+p_B)$ $R_B=\frac{L}{6}(p_A+2p_B)$	$0,125PL\leqslant$ $\leqslant M_{max}\leqslant 0,128PL$	$0,01302\frac{PL^3}{EI}\leqslant$ $\leqslant f_{max}\leqslant$ $\leqslant 0,01304\frac{PL^3}{EI}$	$0,03889\frac{PL^2}{EI}\leqslant$ $\leqslant\varphi_B\leqslant 0,4444\frac{PL^2}{EI}$ idem pour φ_A

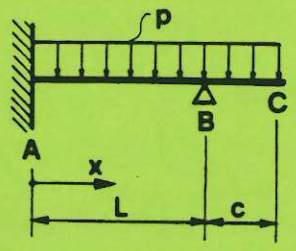
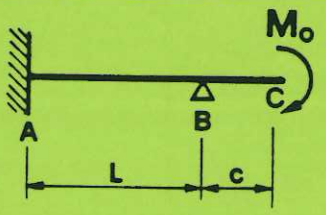
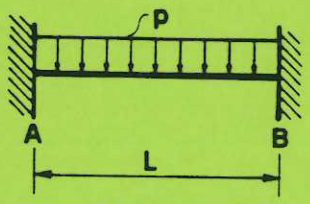
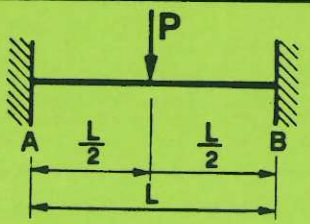
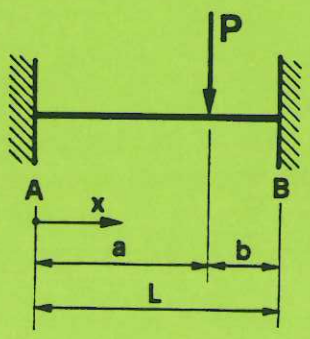
POUTRE À DEUX APPUIS AVEC PORTE-À-FAUX

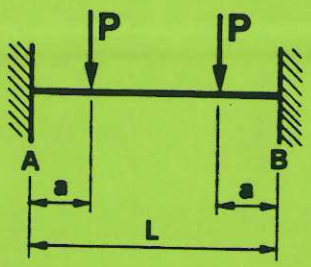
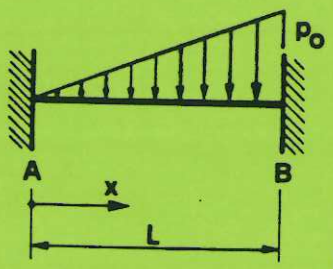
Cas de charge	Réactions d'appui	Moments fléchissants	Flèches	Angles de rotation
	$R_A = -\frac{Pc}{L}$ $R_B = \frac{P(L+c)}{L}$	$M_B = -Pc$	$f_C = \frac{Pc^2(L+c)}{3EI}$	$\varphi_A = \frac{PcL}{6EI}$ $\varphi_B = -\frac{PcL}{3EI}$ $\varphi_C = -\frac{Pc(2L+3c)}{6EI}$
	$R_A = -\frac{pc^2}{2L}$ $R_B = \frac{pc(2L+c)}{2L}$	$M_B = -\frac{pc^2}{2}$	$f_C = \frac{pc^3(4L+3c)}{24EI}$	$\varphi_A = \frac{pc^2L}{12EI}$ $\varphi_B = -\frac{pc^2L}{6EI}$ $\varphi_C = -\frac{pc^2(L+c)}{6EI}$
	$R_A = -\frac{M_0}{L}$ $R_B = \frac{M_0}{L}$	$M_B = -M_0$	$f_C = \frac{M_0c(2L+3c)}{6EI}$	$\varphi_A = \frac{M_0L}{6EI}$ $\varphi_B = -\frac{M_0L}{3EI}$ $\varphi_C = -\frac{M_0(L+3c)}{3EI}$
	$R_A = \frac{p(L^2-c^2)}{2L}$ $R_B = \frac{p(L+c)^2}{2L}$	$M_B = -\frac{pc^2}{2}$	$f_C = \frac{pc(L+c)}{24EI} \times$ $\times(3c^2+cL-L^2)$	$\varphi_A = -\frac{pL(L^2-2c^2)}{24EI}$ $\varphi_B = \frac{pL(L^2-4c^2)}{24EI}$ $\varphi_C = \frac{p}{24EI} \times$ $\times(L^3-4c^3-4c^2L)$
	$R_A = R_B = P$	$M_A = M_B = -Pc$	$f_C = \frac{Pc^2(3L+2c)}{6EI}$ $f_{(\frac{L}{2})} = -\frac{PcL^2}{8EI}$	$\varphi_A = -\varphi_B = \frac{PcL}{2EI}$ $\varphi_C = -\frac{Pc(L+c)}{2EI}$
	$R_A = R_B = \frac{p(L+2c)}{2}$	$M_A = M_B = -\frac{pc^2}{2}$ $M_{(\frac{L}{2})} = \frac{p(L^2-4c^2)}{8}$	$f_C = \frac{pc^4}{24EI} \times$ $\times\left(3+6\frac{L}{c}-\frac{L^3}{c^3}\right)$ $f_{(\frac{L}{2})} = \frac{pL^2}{384EI} \times$ $\times(5L^2-24c^2)$	$-\varphi_A = \varphi_B =$ $= \frac{pL(L^2-6c^2)}{24EI}$ $\varphi_C = \frac{p}{24EI} \times$ $\times(L^3-4c^3-6c^2L)$

POUTRE ENCASTRÉE				
Cas de charge	Réactions d'appui	Moments fléchissants	Flèches	Angles de rotation
	$R_A = P$	$M_A = -PL$	$f_B = \frac{PL^3}{3EI}$	$\varphi_B = -\frac{PL^2}{2EI}$
	$R_A = P$	$M_A = -Pa$	$f_B = \frac{Pa^2(3L-a)}{6EI}$	$\varphi_B = -\frac{Pa^2}{2EI}$
	$R_A = pL$	$M_A = -\frac{pL^2}{2}$	$f_B = \frac{pL^4}{8EI}$	$\varphi_B = -\frac{pL^3}{6EI}$
	$R_A = P$	$M_A = -PL$	$f_B = \frac{PL^3}{3EI_1}(1+n\alpha^3)$	$\varphi_B = -\frac{PL^2}{2EI_1} \times (1+n\alpha^2)$
	$R_A = pL$	$M_A = -\frac{pL^2}{2}$	$f_B = \frac{pL^4}{8EI_1}(1+n\alpha^4)$	$\varphi_B = -\frac{pL^3}{6EI_1} \times (1+n\alpha^3)$
	$R_A = 0$	$M_A = -M_0$	$f_B = \frac{M_0 L^2}{2EI}$	$\varphi_B = -\frac{M_0 L}{EI}$
	$R_A = 0$	$M_A = -M_0$	$f_B = \frac{M_0 a(2L-a)}{2EI}$	$\varphi_B = -\frac{M_0 a}{EI}$

Cas de charge	Réactions d'appui	Moments fléchissants	Flèches	Angles de rotation
	$R_A = 0$	$M_A = -M_0$	$f_B = \frac{M_0 L^2}{2EI_1}(1+n\alpha^2)$	$\varphi_B = -\frac{M_0 L}{EI_1}(1+n\alpha)$
	$R_A = pa$	$M_A = -\frac{pa^2}{2}$	$f_B = \frac{pa^3(4L-a)}{24EI}$	$\varphi_B = -\frac{pa^3}{6EI}$
	$R_A = \frac{p_0 L}{2}$	$M_A = -\frac{p_0 L^2}{6}$	$f_B = \frac{p_0 L^4}{30EI}$	$\varphi_B = -\frac{p_0 L^3}{24EI}$
	$R_A = \frac{p_0 L}{2}$	$M_A = -\frac{p_0 L^2}{3}$	$f_B = \frac{11 p_0 L^4}{120EI}$	$\varphi_B = -\frac{p_0 L^3}{8EI}$
POUTRE ENCASTRÉE - APPUYÉE				
	$R_A = \frac{3}{8}pL$ $R_B = \frac{5}{8}pL$	$M_B = -\frac{pL^2}{8}$ $M^{\oplus}_{max} = \frac{9pL^2}{128}$ pour $x = \frac{3}{8}L$	$f_{max} \approx \frac{pL^4}{185EI}$ pour $x = 0,4215L$	$\varphi_A = -\frac{pL^3}{48EI}$
	$R_A = \frac{5}{16}P$ $R_B = \frac{11}{16}P$	$M_B = -\frac{3}{16}PL$ $M_P = \frac{5}{32}PL$	$f_P = \frac{7PL^3}{768EI}$ $f_{max} = \frac{PL^3}{48\sqrt{5}EI}$ pour $x = \frac{L}{\sqrt{5}}$	$\varphi_A = -\frac{PL^2}{32EI}$

Cas de charge	Réactions d'appui	Moments fléchissants	Flèches	Angles de rotation
	$R_A = \frac{Pb^2(3L-b)}{2L^3}$ $R_B = \frac{Pa(3L^2-a^2)}{2L^3}$	$M_B = -\frac{Pa(L^2-a^2)}{2L^2}$ $M_P = \frac{Pab^2(3L-b)}{2L^3}$	$f_P = \frac{Pa^2b^3(3L+a)}{12EIL^3}$ $f_{max}=f_P$ quand a = $= (\sqrt{2}-1)L=0,414L$ si $a \leqslant 0,414L$: $f_{max} = \frac{Pab^2}{6EI}\sqrt{\frac{a}{2L+a}}$ en $x = L\sqrt{\frac{a}{2L+a}}$ si $a \geqslant 0,414L$: $f_{max} = \frac{Pa}{3EI} \times$ $\times \frac{(L^2-a^2)^3}{(3L^2-a^2)^2}$ en $x = L\frac{L^2+a^2}{3L^2-a^2}$	$\varphi_A = -\frac{Pab^2}{4EIL}$
	$R_A = -\frac{3}{2}\frac{M_0}{L}$ $R_B = \frac{3}{2}\frac{M_0}{L}$	$M_A = M_0$ $M_B = -\frac{1}{2}M_0$	$f_{max} = \frac{M_0L^2}{27EI}$ pour $x = \frac{1}{3}L$	$\varphi_A = -\frac{M_0L}{4EI}$
	$R_A = \frac{p_0L}{10}$ $R_B = \frac{2p_0L}{5}$	$M_B = -\frac{p_0L^2}{15}$ $M^{\oplus}_{max} = \frac{p_0L^2}{15\sqrt{5}} \cong$ $\cong \frac{3p_0L^2}{100}$ pour $x = \frac{L}{\sqrt{5}}$	$f_{max} = \frac{2p_0L^4}{375\sqrt{5}EI} \cong$ $\cong \frac{p_0L^4}{420EI}$ en $x = \frac{L}{\sqrt{5}}$	$\varphi_A = -\frac{p_0L^3}{120EI}$
POUTRE ENCASTRÉE - APPUYÉE AVEC PORTE-À-FAUX				
	$R_A = -\frac{3Pc}{2L}$ $R_B = \frac{P(2L+3c)}{2L}$	$M_A = \frac{Pc}{2}$ $M_B = -Pc$	$f_C = \frac{Pc^2(3L+4c)}{12EI}$	$\varphi_B = -\frac{PcL}{4EI}$ $\varphi_C = -\frac{Pc(L+2c)}{4EI}$

Cas de charge	Réactions d'appui	Moments fléchissants	Flèches	Angles de rotation
	$R_A = \frac{p(5L^2-6c^2)}{8L}$ $R_B = \frac{p}{8L} \times$ $\times(3L^2+8cL+6c^2)$	$M_A = -\frac{p(L^2-2c^2)}{8}$ $M_B = -\frac{pc^2}{2}$ $M^{\oplus}_{max} = \frac{9pL^2}{128} +$ $+ \frac{pc^2}{32}\left(9\frac{c^2}{L^2} - 7\right)$ en $x = \frac{5L^2-6c^2}{8L}$	$f_C = \frac{pc}{48EI} \times$ $\times(6c^3+6c^2L-L^3)$	$\varphi_B = \frac{pL}{48EI} \times$ $\times (L^2-6c^2)$ $\varphi_C = \frac{p}{48EI} \times$ $\times(L^3-6c^2L-$ $-8c^3)$
	$R_A = -\frac{3}{2}\frac{M_0}{L}$ $R_B = \frac{3}{2}\frac{M_0}{L}$	$M_A = \frac{1}{2} M_0$ $M_B = -M_0$	$f_C = \frac{M_0 c(L+2c)}{4EI}$	$\varphi_B = -\frac{M_0 L}{4EI}$ $\varphi_C = -\frac{M_0(L+4c)}{4EI}$
POUTRE BI-ENCASTRÉE				
	$R_A = R_B = \frac{pL}{2}$	$M_A = M_B = -\frac{pL^2}{12}$ $M^{\oplus}_{max} = M(\frac{L}{2}) = \frac{pL^2}{24}$	$f_{max} = f(\frac{L}{2}) = \frac{pL^4}{384EI}$	
	$R_A = R_B = \frac{P}{2}$	$M_A = M_B = -\frac{PL}{8}$ $M_P = M(\frac{L}{2}) = \frac{PL}{8}$	$f_P = \frac{PL^3}{192EI}$	
	$R_A = \frac{P}{L^3} \times$ $\times (L-a)^2(L+2a)$ $R_B = \frac{Pa^2(3L-2a)}{L^3}$	$M_A = -\frac{Pa(L-a)^2}{L^2}$ $M_B = -\frac{Pa^2(L-a)}{L^2}$ $M_P = \frac{2Pa^2(L-a)^2}{L^3}$	$f_P = \frac{Pa^3(L-a)^3}{3EIL^3}$ si a>b: $f_{max} = \frac{2Pa^3b^2}{3EI(L+2a)^2}$ en $x = \frac{2aL}{L+2a}$ si a<b: $f_{max} = \frac{2Pa^2(L-a)^3}{3EI(3L-2a)^2}$ en $x = \frac{L^2}{3L-2a}$	

Cas de charge	Réactions d'appui	Moments fléchissants	Flèches	Angles de rotation
	$R_A = R_B = P$	$M_A = M_B = -\frac{Pa(L-a)}{L}$ $M_P = M_{(\frac{L}{2})} = \frac{Pa^2}{L}$	$f_P = \frac{Pa^3(2L-3a)}{6EIL}$ $f_{max} = f_{(\frac{L}{2})} = \frac{Pa^2(3L-4a)}{24EI}$	
	$R_A = \frac{3p_0L}{20}$ $R_B = \frac{7p_0L}{20}$	$M_A = -\frac{p_0L^2}{30}$ $M_B = -\frac{p_0L^2}{20}$ $M^{\oplus}_{max} = \frac{p_0L^2}{30}(3\sqrt{\frac{3}{10}}-1) = \frac{p_0L^2}{46,65}$ pour $x = L\sqrt{\frac{3}{10}}$	$f_{max} = \frac{p_0L^4}{764EI}$ pour $x = 0,525L$	